Capacitance Review Sheet

AP Physics, G Period
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Background

Capacitance is a measure of charge stored on a conductor at a given electric potential. This sheet will review combinations of capacitors, dielectrics, and important formulas.

Formulas

$C = \frac{Q}{V}$ definition of capacitance

$C = \varepsilon_0 \frac{A}{d}$ capacitance of a parallel plate

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$ equivalent capacitance in series

$C_{eq} = C_1 + C_2 + \dots$ equivalent capacitance in parallel

$C = \kappa \varepsilon_0 \frac{A}{d}$ capacitance of parallel plate with dielectric

$U = \frac{1}{2} C V^2$
$\frac{1}{2} Q V$ electric potential energy stored in capacitor

Key concepts/terms

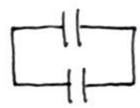
Capacitance is a positive value that measures a conductor's ability to hold charge with the unit Farads. By definition, it is. $\frac{Q}{V} = \frac{\text{Coulombs}}{\text{Volts}}$

How to find capacitance:

1. Assume charges
2. Find the electric potential difference, $V = \int E \cdot dr$
   a. Use Gauss's Law to determine the electric field, $\int E \cdot dA = \frac{q_{enclosed}}{\varepsilon_0}$
3. Plug information into the definition of capacitance, $C = \frac{Q}{V}$

Capacitors in series
Capacitors in series share common charge, q

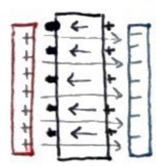
Capacitors in parallel
Capacitors in parallel have a common voltage drop, V. They are connected with a neighbor in two places.



Dielectrics are insulating materials that increase capacitance.

Thus, $C_{\text{with dielectric}} = \kappa C_{\text{without}}$

The dielectric constant, κ varies for different materials. It is always greater than 1.



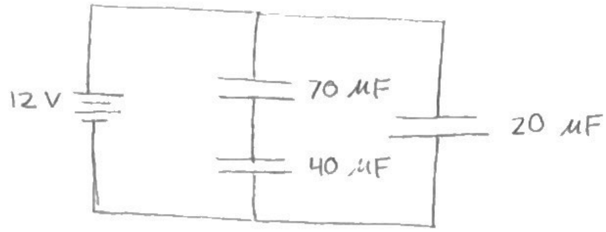
vander Waal's effect creates a weak E-fld in the opposite direction (purple).

↓ E-fld = ↓ V

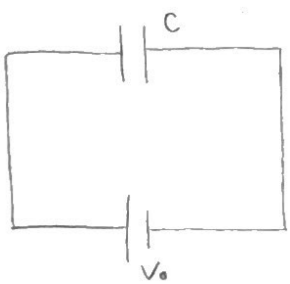
★ $C = \frac{Q}{V}$, so C increases

Problems

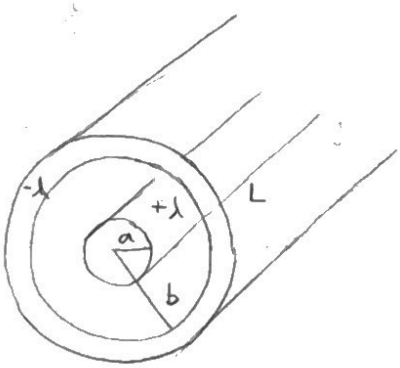
1. **Easy:** Find the equivalent capacitance of the system.



2. **Medium:** A simple circuit is illustrated below. What happens to the capacitance, voltage, charge, electric field, and energy in each situation? Does it increase, decrease, or stay the same?
    a. A dielectric is placed in the capacitor.
    b. The plates of the capacitor are pulled further apart.
    c. The plates of the capacitor are exchanged for larger plates.

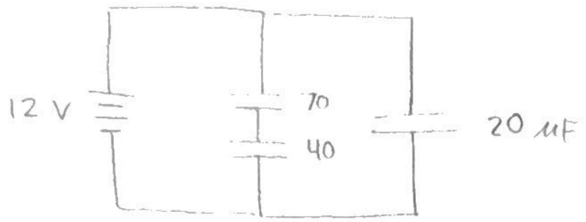


3. **Hard:** Determine the capacitance of the coaxial cable shown below. Then, the capacitor is connected to a battery with potential difference, 3V. What is the maximum energy the capacitor can store?



Solutions

1. Easy



The 70 and 40 μF capacitors are in series, and they are both in parallel with the 25 μF capacitor.

First, we find C_{eq} of the two in series: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

$$C_{eq} = \left(\frac{1}{70} + \frac{1}{40}\right)^{-1}$$

$$C_{eq} = 25.45\ \mu F$$

Now, we can find C_{eq} of the system using the capacitors in parallel relationship.

$$C_{eq} = C_1 + C_2$$

$$C_{eq} = 25.45 + 20 = \boxed{45.45\ \mu F}$$

2. Medium

a. Dielectric

Capacitance - increases by formula $C = k\varepsilon_0 \frac{A}{d}$ since k is always > 1. Voltage stays the same and is supplied by the battery. Charge increases by $C = \frac{Q}{V}$; C increases, V stays the same, so Q must also increase. Electric field stays the same since $E = \frac{V}{d}$ and V and d don't change. Energy increases since $U = \frac{1}{2}CV^2$ and C increases.

b. Plates pulled apart

Capacitance decreases since $C = k\varepsilon_0 \frac{A}{d}$ and d increased. Voltage stays the same since the battery is still connected. Charge decreases by $C = \frac{Q}{V}$; C decreases, V is the same, so Q also decreases. Electric field decreases since $E = \frac{V}{d}$ and d increased. Energy decreases since $U = \frac{1}{2}CV^2$ and C decreased.

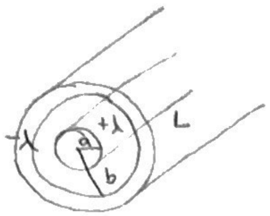
c. Larger plates

Capacitance increases since $C = k\varepsilon_0 \frac{A}{d}$ and A increased. Voltage stays the same - the battery is still connected. Charge increases by $C = \frac{Q}{V}$; C increases, V is the same, so Q also increases. Electric field stays the same since $E = \frac{V}{d}$ and neither V nor d changed. Energy increases since $U = \frac{1}{2}CV^2$ and C increased.

3. Hard

We need to use the approach here.

First, we use Gauss's Law to determine the E-fld.



$$\int E \cdot dA = \frac{q_{enc}}{\epsilon_0}$$

$$E(2\pi r L) = \frac{+\lambda L}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi r \epsilon_0}$$, E-fld is outward

Next, we can integrate the E-fld to get the potential difference.

$$V = \int E \cdot dr$$

$$V = \int_{r=a}^{r=b} \frac{\lambda}{2\pi r \epsilon_0} dr$$

Because the E-fld is outward, we integrate from inside to outer (a to b).

$$V = \frac{\lambda}{2\pi\epsilon_0} \int_a^b \frac{1}{r} dr$$

$$V = \frac{\lambda}{2\pi\epsilon_0} \ln r \Big|_a^b$$

$$V = \frac{\lambda}{2\pi\epsilon_0} \ln b - \ln a = \frac{\lambda}{2\pi\epsilon_0} \ln\frac{b}{a}$$

Now, we can plug stuff in to the definition

$$C = \frac{Q}{V}$$

$$C = \frac{\lambda L}{\frac{\lambda}{2\pi\epsilon_0}\ln\frac{b}{a}} = \boxed{\frac{2\pi\epsilon_0 L}{\ln\frac{b}{a}}}$$

To find electric potential energy:

$$U = \frac{1}{2} C V^2$$

$$U = \frac{1}{2}\left(\frac{2\pi\epsilon_0 L}{\ln\frac{b}{a}}\right)(3V)^2$$

$$U = \frac{9}{2}\left(\frac{\cancel{2\pi\epsilon_0} L}{\cancel{\ln\frac{b}{a}}}\right)\left(\frac{\lambda}{\cancel{2\pi\epsilon_0}} \cancel{\ln\frac{b}{a}}\right)^2$$

$$\boxed{U = \frac{9}{2} \frac{\lambda^2 L}{2\pi\epsilon_0} \ln\frac{b}{a}}$$